Algebraic Reasoning in the Middle Grades:
A View of Student Strategies in Pictorial and Algebraic System of Equations

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Abstract

Teachers use action research in order to improve their teaching and student learning. This action research will analyze student's algebraic reasoning in finding values of variables in systems of equations pictorially and algebraically. This research will look at students solving linear systems of equations without knowing the algebraic algorithms. It will concentrate on the students constructive learning abilities to resolve and create methods in solving linear systems of equations. The study will take a look at several questions. What strategies and methods do students' use to solve for variables in a pictorial system of equations? What language do students' develop in their understanding of the equation sign? Are students more successful in solving linear systems pictorially or algebraically? The participants are 8th graders in a remedial summer course from a southwest middle school. Participants were given a pre-test and a post-test to determine the amount of increase or decrease in scores. The results show students actually decreased in scores.

Algebraic Reasoning in the Middle Grades:
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Algebraic reasoning in the middle grades is important to facilitate and provide a foundation for success in the higher grades. This action research will look at the strategies middle school students use in algebraic reasoning It will provide a series of intervention activities to help promote and improve students' algebraic reasoning. The participants have difficulty in passing a state assessment and were required to attend summer school. These students are low performing according to states standards but can perform at a level of conceptual understanding of high level mathematics usually reserved for the high school grades. The paper will also include more information on the purpose, the participants, and methodology of the study, data, results, and a discussion for further research.

Purpose

Students in the Middle School Grades have the potential for participating in higher level mathematics. This action research will examine students' strategies in solving pictorial and algebraic linear systems of equations that is usually reserved for Algebra 1 students. This research will look at students solving linear systems of equations without knowing the algebraic algorithms. It will concentrate on the students constructive learning abilities to resolve and create methods in solving linear systems of equations. The study will take a look at several questions. What strategies and methods do students' use to solve for variables in a pictorial system of equations? What language do students' develop in their understanding of the equation sign? Are students more successful in solving linear systems pictorially or algebraically?

Students in the middle grades have difficulty in algebraic reasoning due to insufficient lessons created by textbooks and district curriculums. This action research will hope to improve students understanding of algebraic reasoning, variables, and the equation sign through the problem solving activities of linear systems in pictorial and algebraic forms. The study focuses on activities that will facilitate students' self constructed knowledge of solving linear systems pictorially and algebraically.

Participants

The participants in the study are from a west Texas middle school near a large border area near Mexico. The students are summer school participants who are required to attend based on criteria set by district or state mandates. These mandates can include anything from poor attendance, failure to pass one or more state assessments, and failing a semester of core classes (math or reading.) All participants in the study failed the math portion of the Texas Assessment of Knowledge and Skills (TAKS) during the first and second administration of the test. These students are required to pass the third administration of the test for promotion to the high school. The participants (n=9) are predominately female and including one male. All students have passed the requirements in their math courses during the school year.

Methods

The study takes place over two class periods that last two hours. The students participate in a warm-up activity, pre-test, problem solving activities, a reflection, and a post-test. The warm-up activity has students simply draw any number of stars and circles then the teacher gives

each symbol a value. Students now find the total values of all the stars and circles combined together. The pre-test consists of four problems. The first question asks student to solve a pictorial system of two equations. The second asks students to solve the same type of problem set up algebraically. Question number three asks students to solve a pictorial system of three equations containing three symbols. Here, the students see three equations including a football, soccerball, and a baseball. Each equation in the system contains two balls equaling a specific weight. The last question asks student to solve an algebraic linear systems of three equations.

The intervention activities are set up to promote the conceptual mathematics of linear systems of two equations. The activities develop the construction of student's knowledge of how to find the values of the pictures or variables. First students examine the balance scale with combination of two figures. Students then determine which figure weighs the most and the least. Students convert the pictorial representation algebraically. Students then examine a pictorial linear system of two equations and determine the process of dissecting the values of each figure. Immediately afterwards, students then write the pictorial form of the linear equations in algebraic form. Student then solve an algebraic system of equations with two variables.

The activities begin to help students make sense of a linear system of three equations with three variables first pictorially then algebraically. Historically, students in higher level of mathematic classes solve linear systems of three equations with three variables by substitutions, combining equations, elimination, and matrices. These activities help students construct their own methods in determining the value of the three figures or variables.

Data

Table 1 and Table 2 show the students performance on the pre-test and the post-test in categories of pictorial and algebraic linear systems.

Table 1. Pre-test scores.

Survey Number	Gender Code*	Ethnic Code**	Time		Pictorial two variables	Algebraic two variables	Pictorial three variables	Algebraic three variables	Score***
			min	sec					
1	2	2	8	4	1	1	1	1	4
2	2	1	4	47	1	0	1	1	3
3	2	1	6	45	1	0	1	0	2
4	2	1	4	42	1	0	1	0	2
5	2	1	7	15	1	0	1	1	3
6	2	1	9	22	0	1	1	1	3
7	2	1	8	30	0	1	1	1	3
8	1	1	n/a	n/a	1	0	0	0	1
9	2	1	8	30	1	1	1	1	4
Totals			54	235	7	4	8	6	25
Averages			6.0	26.1	0.8	0.4	0.9	0.7	2.8

* Gender code represents 1 for males and 2 for females.
** Ethnic code represents 1 for Hispanic and 2 for Anglo.
***Scores are compiled by adding all four categories. A 1 is given to correct answers and a 0 for incorrect answers.

Table 2. Post-test scores

Survey Number	Gender Code*	Ethnic Code**	Time		Pictorial two variables	Algebraic two variables	Pictorial three variables	Algebraic three variables	Score*
			min	sec					
1	2	2	20	15	1	0.5	0	1	2.5
2	2	1	30	0	0	1	0	0	1

3	2	1	25	46	1	0	0	0	1
4	2	1	23	31	1	1	0	0	2
5	2	1	25	26	0.5	1	0	0	1.5
6	2	1	29	27	1	1	0	1	3
7	2	1	n/a	n/a	1	1	0	1	3
8	1	1	n/a	n/a	1	0	0	0	1
9	2	1	n/a	n/a	1	1	0	1	3
Totals			152	145	7.5	6.5	0	4	18
Averages			16.9	16.1	0.8	0.7	0.0	0.4	2.0

* Gender code represents 1 for males and 2 for females.
** Ethnic code represents 1 for Hispanic and 2 for Anglo.
***Scores are compiled by adding all four categories. A 1 is given to correct answers and a 0 is given for incorrect answers.

Figure 1. Chart of pre-test and post-test scores.

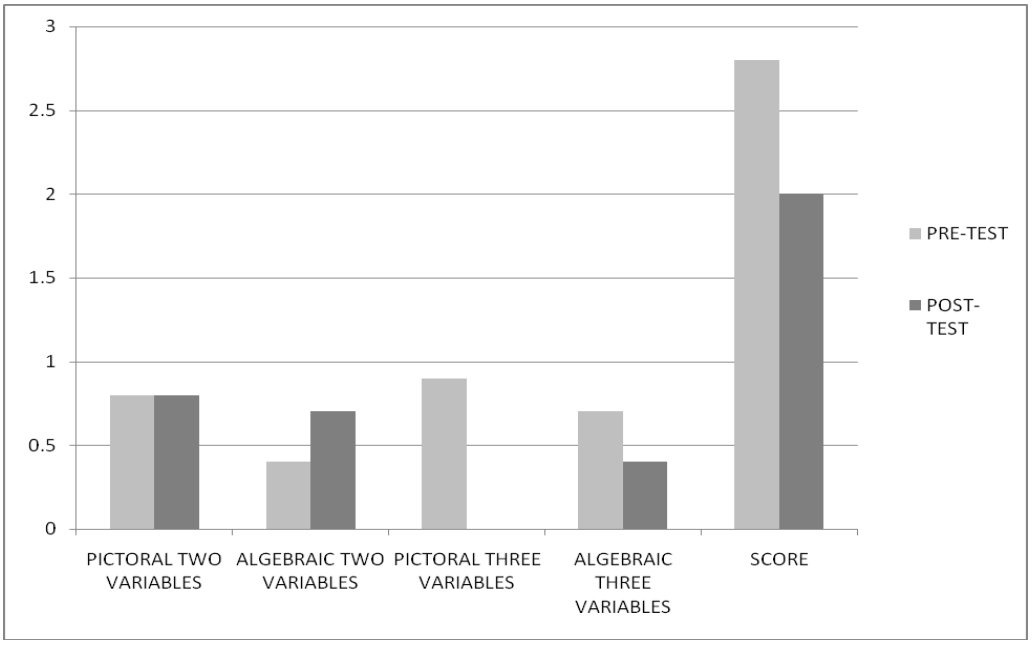


Figure 1 displays the scores of the participants from the pre and post-tests.

Figure 2 shows the difference in completion times between the two tests.

Figure 2. Average pre and post-test completion times.

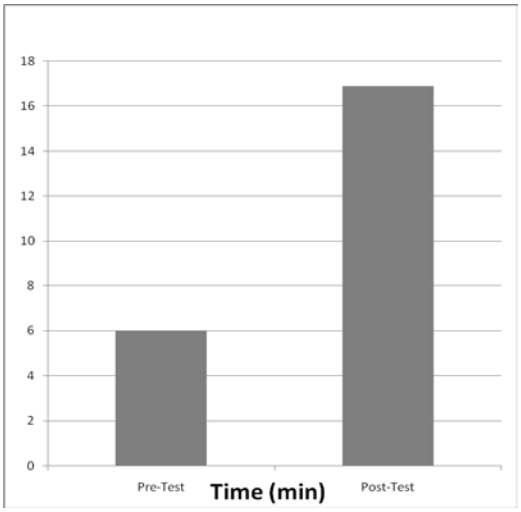


Table 3 shows the strategies students use in the pre and post-tests including: combining equations, substitution, did not complete the problem, and the usage of values in variables.

Table 3. Strategies in pre and post-tests.

	Pre-test	Post-test
Did not complete	0	1
Guess and Check	30	17
Convert to algebraic form	0	2
Symbols have same value	1	0
One variable has many values	5	1
Combines equations	0	4
Uses substitution	0	8

Figure 3 displays the strategies students use in the pre and post-tests.

Figure 3. Strategies in pre and post-tests.

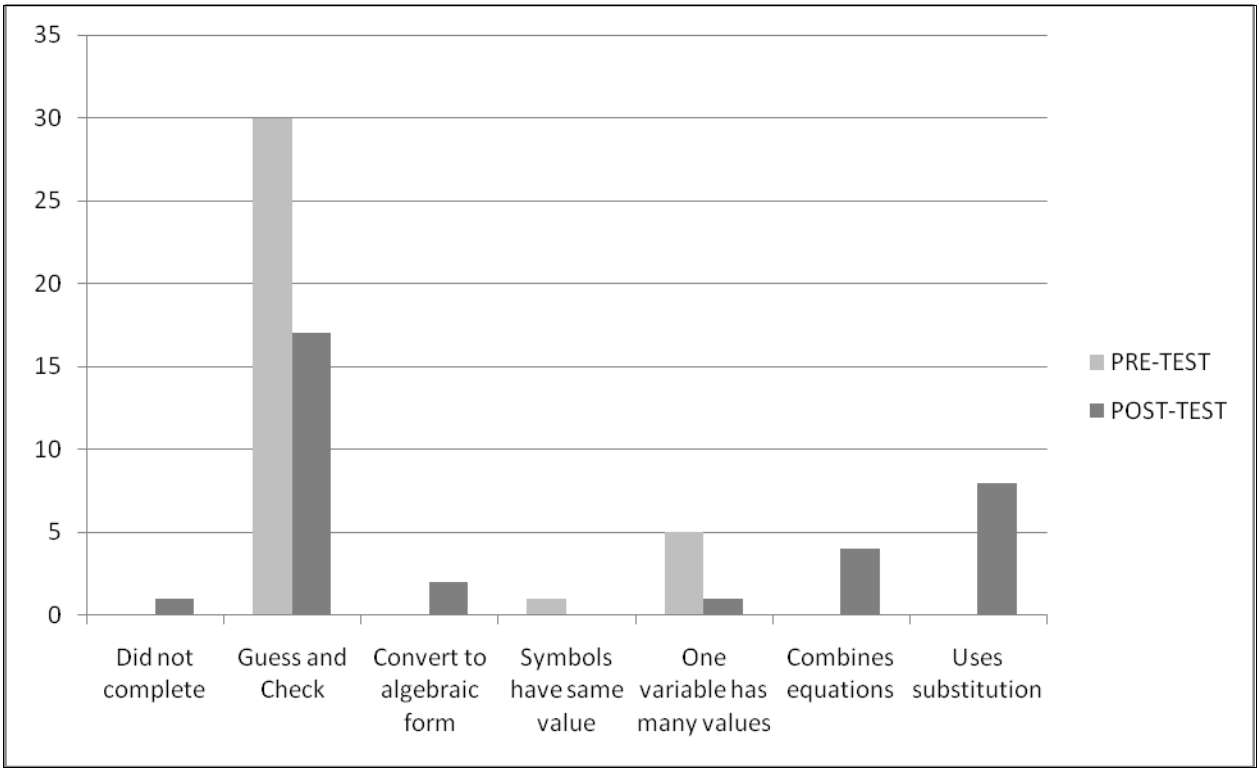
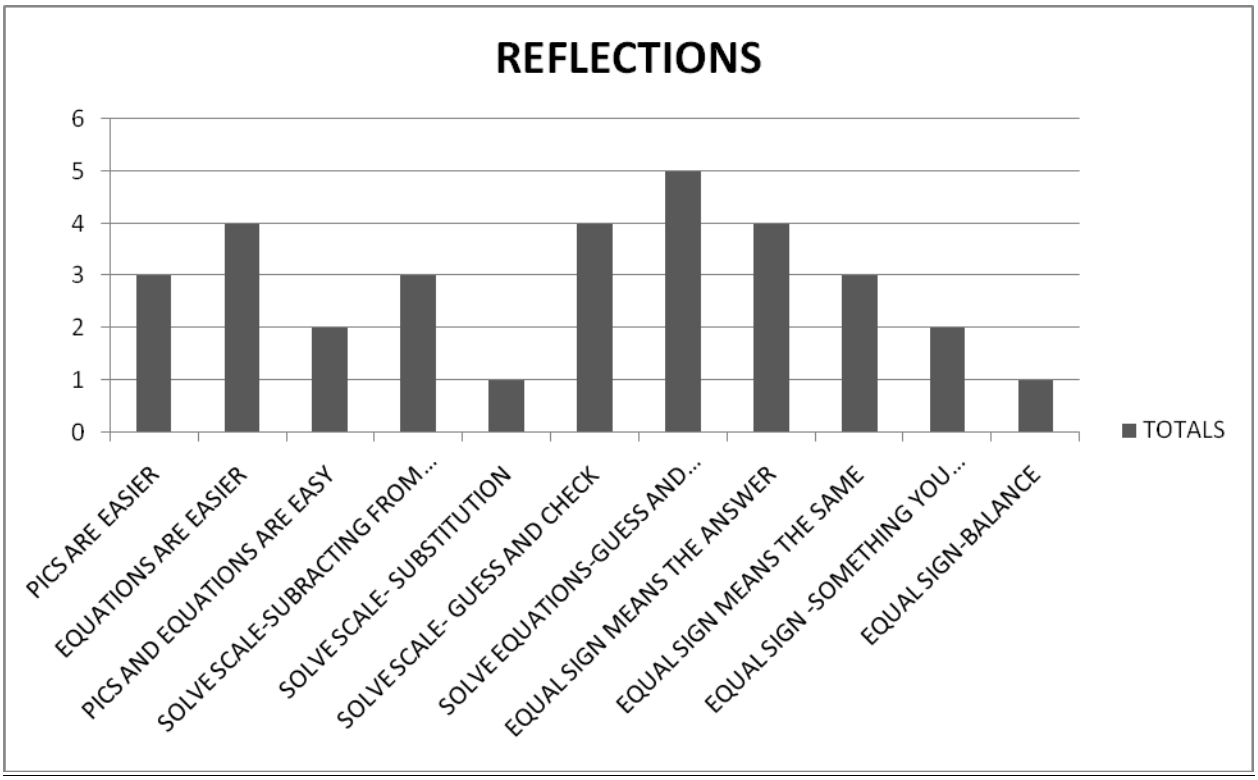


Figure 4 shows the strategies students wrote in their reflections. Questions included are: What strategies did you use? Which are easier pictures or variables in solving the systems of equations? What did you think the equal sign represents?

Figure 4. Reflections on strategies.



Results

The most significant result from the data is displayed in Figure 1. Students decreased in their overall scores from the pre-test to the post-test. The chart also displays how each type of problem increased or decreased in each of the tests. The data in Figure 2, displaying the completion times, shows a significant increase in the time spent on the post-test compared to the pre-test. This is a result of students using the strategies they develop in the activities. Figure 3 compares the strategies in both tests. Students mostly use the guess and check method in the pretest, however; it loses ground in the post-test.

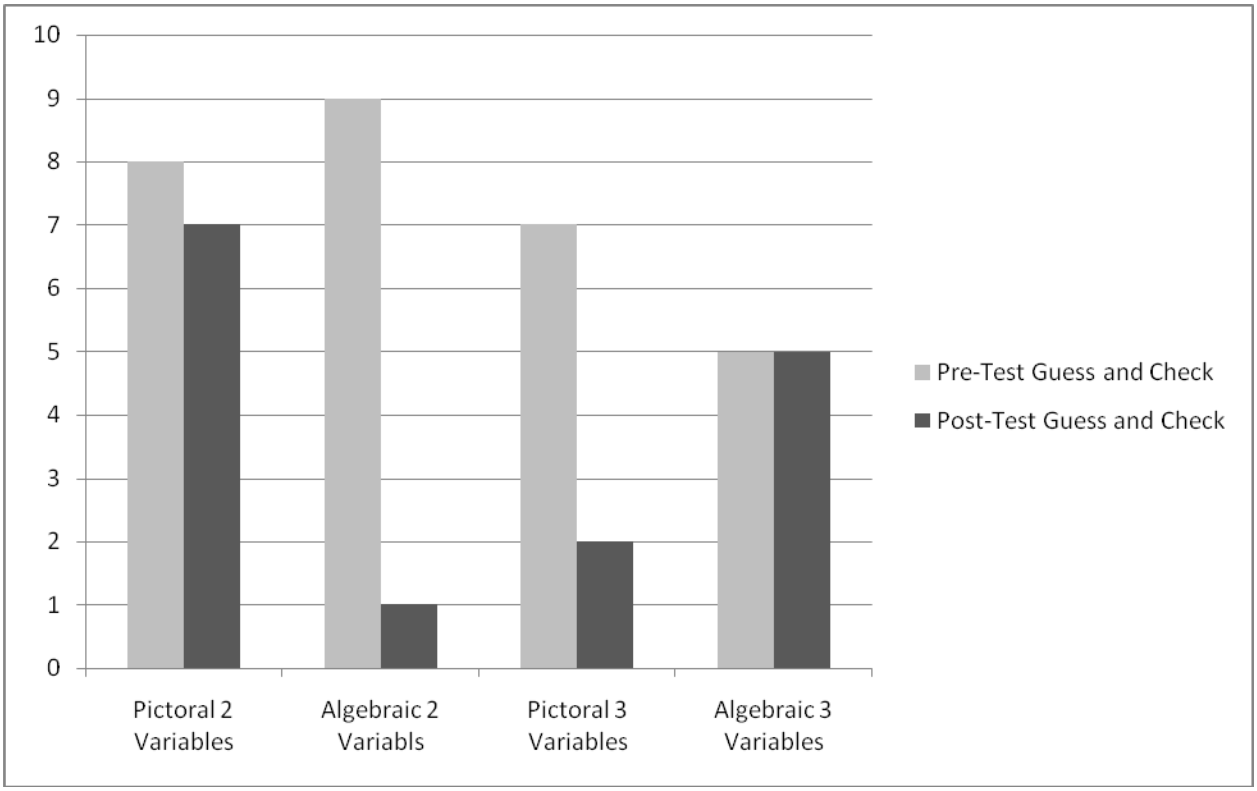
Discussion

The most significant piece of data that is very revealing but deceptive is the drop in scores. This may be a factor of how different the tests are in difficulty. Much of the pre-test is

comprised of easy problems where students can merely use guess and check to find the solutions to the systems of equations. The post-test three variable algebraic linear system contains decimals that gave students a difficult time in using the guess and check method.

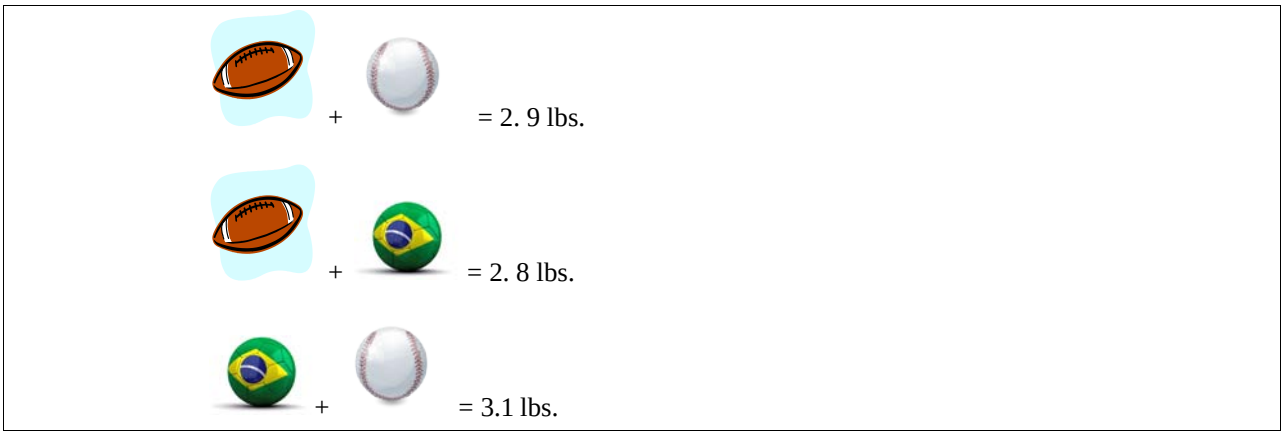
Although we see in the pre-test the wide use of guess and check, there are gains in the post-test in strategies students use and a decrease of the guess and check method. In the pre-test, students use the guess and check strategy thirty times in the test compared to seventeen times in the post-test. This is a result of students trying the methods they develop during the intervention activities. Figure 5 displays the amount of students who use the guess and check method within the problems of both tests.

Figure 5. Number of guess and check strategies



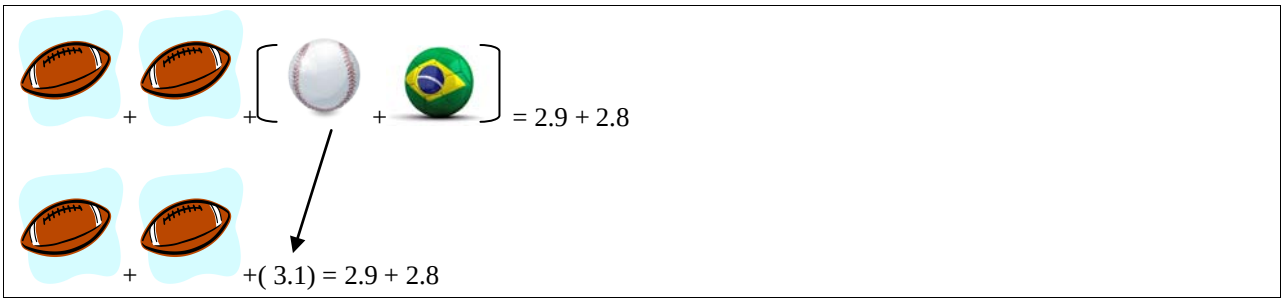
Other strategies begin to appear in the post-test where in fact they were in absent in the pre-test. These strategies include combining the equations together and substitutions (see Figure 3.) In combining the equations together, students did the following for solving a linear system with three equations pictorially in Figure 6a.

Figure 6a. Example of student algebraic reasoning.



Here students add the top two equations together and see they can substitute the quantity in the parenthesis with the quantity from the third equation (Figure 6b.)

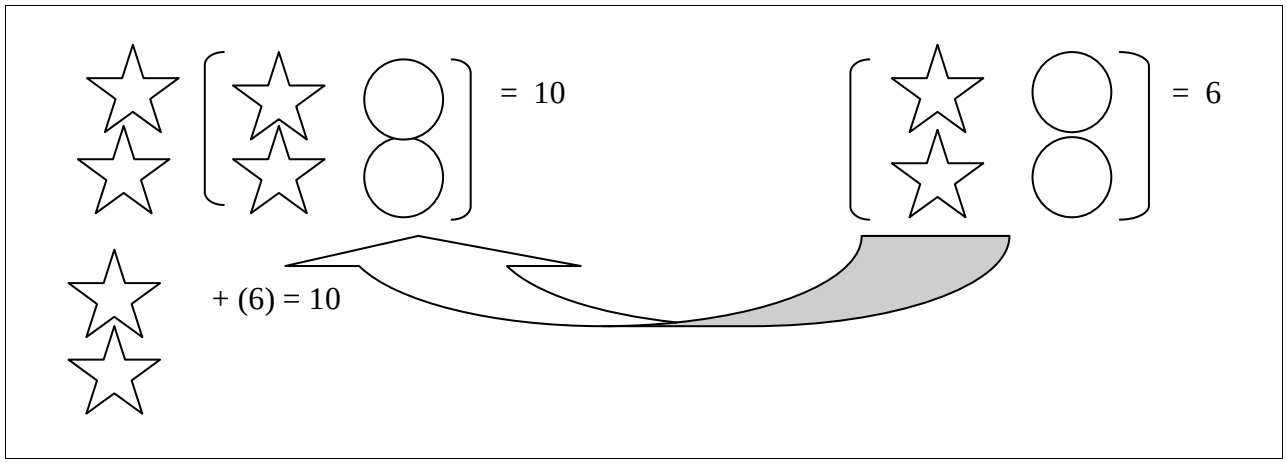
Figure 6b. Example of student algebraic reasoning.



Students first solve for the football then use substitution to find the other values of the remaining objects. Students use the method in the post-test but forget to combine the quantities from the right side of the equations. Even though students got the problem incorrect(since this problem provides difficulty in using the guess and check method,) they understood the conceptual meaning of the abiltity to combine equations together.

Students also use substitution when solving a system of two equations with two variables (Figure 7.)

Figure 7. Student strategies in pictorial systems.



Students can see that two stars have to equal four so each star equals two.

A problem in the pre-test is where five students gave different values to the same variable in order to solve the systems, i.e., x =4 and then x =5. In the post-test, only one student repeated the behavior. This shows that students now understand the value of the variable must be the same in all equations. Students also see that substitution works in the algebraic form as well when they solve for one variable. Students solve the systems of two equations algebraically by solving for one variable and the substituting to find the value of the other missing variables. (Figure 8.)

Figure 8. Strategies in algebraic reasoning in systems using substitution.

$$\begin{cases} 2x + 4y = 14 \\ 4y = 12 \end{cases}$$

$4y=12$

$y=3$, then

$2x + 4(y) = 14$

$2x + 4(3) = 14$

$2x + 12 = 14$

$2x = 2$

$x=1$

Students develop the substitution strategy in the intervention activities. Seven out of the nine students were able to apply the substitution method in the post-test. Some students applied the concept but had difficulty in solving for the missing value.

In their written reflections, student have mixed feelings about whether pictorial representations or algebraic equation linear systems are easier (Table 4.)

Table 4. Pictorial vs. Algebraic

Pictorial	3
Equations	4
Both	2

N=9

Students also did not develop a good concept of the equal sign. Only one student understood that the equal sign represents a balance between both sides of the equations. Most students have the misconception of what the equation sign represents. The examples and discussion of the equal sign as a balance did not interest them at all. In their reflections, they are

firm believers that the equal sign either represents the answer or something you solve for in an equation. Undoing this misconception will take further lessons and hands-on activities.

Conclusion

Scores do not reflect the amount of learning students develop in the action research. The real data is in the student work along with the strategies they develop during the intervention activities. Students know the strategies they develop but sometimes have difficulty in the algorithms of solving algebraic equations. In answering the research questions, the study did address each one. Overall students use strategies they develop in the intervention activities (as also seen in the increase in completion times in the post-test.) Students use substituion and combining equations to solve the systems both algebraically and pictorially. Althought students use the strategies, their lack of conceptual understanding in solving equations hinders them from completing the process to solve a linear system whether containing two or three equations.

Finally, further research is needed. Further research on more students can illuminate the research questions in the study. Changes in the post-test problems to inhibit guess and check strategies can promote the usage of the new strategies students develop. This will improve the data information.